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PYTHAGORAS THEOREM IN ANCIENT INDIAN VASTU TEXTS

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Introduction

Vastuvidya is a traditional science of manifestation. According to Indian Mythology [1] this Sastra, science of manifestation is originated from lord Brahma and is expanded through the saints. Vastuvidya is described in stapathyaveda which is treated as a part of Atharvaveda. The principles and methods of vastuvidya are described in many traditional texts. These texts are generally divided into puranas, samhitas, agamas and modern texts. The original form of stapathyaveda is not available now. But the contents of it like calculations and ideas are now seen on other texts like sulbasutra, kalpa texts etc. The first construction in Vastu with scientific approach is Altars (Vedis). There is a close connection between vastuvidya (traditional science) and modern mathematics. Sulbasutras describe the rules of construction of different types of altars, pandals, and places for sacred fire. The mathematical part of the construction especially the geometrical part is given only in Sulbasutras. Sulbasutras describe the shapes of citis or altars of sacred fires which may differ in shape but their areas will remain the same.

The main Sulbasutras dealing with mathematics are
1. Baudhayana Sulbasutra 2. Apastamha Sulbasutra and 3. Katyayana Sulbasutra.

There are interesting results in Geometry in connection with the measurements of yajnakundas. These works are described in the Sulbasutras which are of the late Sutra period. To attain certain desired objectives, religiously, the vedic Indians performed some Kamya Yajnas according to the vedic tradition. Depending on the nature of sacrifice, specific types of fire altars were to be constructed. All these figures involve a lot of mathematical applications in their calculations of dimensions and geometrical constructions. This reveals the development of mathematical techniques to construct and transform the geometrical figures involved in it. Triangle is the basic figure for all the kundas (square is a combination of two triangles). The transformation geometry of the sulbasutras supports this method.

The Pythagorean Theorem was known in India even at the age of *Subva-sutras*. *Subva-sutra* indirectly gave a proof of the Pythagorean theorem using area considerations. India had made a very large contribution in the field of Mathematics and traditional architecture. This is evaluated by comparing it with modern science.

References of the 'theorem of hypotenuse' in ancient vastu texts

There are evidences that Vedic Aryans had knowledge about geometry. The vedic Aryans gave considerable importance to yajnas. They believed that the correct measurements and orientation ensured the effectiveness of the sacrifice. They had clear knowledge about the correct orientation and measurements of altars and fireplaces and these were codified in Sulbasutras. This knowledge is older than the Sulbasutras. Geometrical constructions employed in the Sulbasutras include the theorem of hypotenuse (Now known as Pythagoras theorem) was used in Sulbasutras and sulbakaras gave a large number of such side-sets. By trying to divide the chord of a given length into parts capable of yielding different right angled triangles, Sulbakaras gave a general solution as $2n(n+1), 2n+1, 2n(n+1)+1$. Another general solution from Katyayanas method for combining any number of equal squares into a big square gave another general solution $m, (m^2-1)/2, (m^2+1)/2$.

For some type of yajnas the shape of the altar was divided into one or more squares. For this the addition or subtraction of squares was necessary. This process was done based on the theorem of hypotenuse. The altar for the Asvamedha sacrifice must be twice or three times the area of the basic altar [2].

The area of the altar to be constructed is either twice or thrice the area of the first basic altar which is of area 7.5 square purusas. The area of the Sautramani vedi is one-third of the area of the Mahavedi [3].The altar for the Pitryajna is to be formed with the one-third part of the side of the Mahavedi and its area will be equal to the one ninth part of the Mahavedi [4].

The area of the Mahavedi is 972 square padas [5]. The area of the Sautramani vedi is 324 square padas. The area of the vedi for Pitryajna will be $1/9 \times 972 = 108$ square padas.

The difference between Pythagoras theorem and theorem of hypotenuse stated by Sulbakaras is that they refer to a rectangle or to a square instead of the right-angled triangle. Even though the reference is made to a rectangle or to a square, their aim was to refer only to the two sides and the diagonal. If their aim was to refer to a rectangle or to a square, they will refer to all the four sides. The mathematical part of the Sulbasutras contains theorems of squares and rectangles. Their aim was to find out the side of a square which is equal in area to the sum or difference of two squares, or to transform a circle into a square or a triangle into a square.

The Sulbakaras did not give any direct proof for the Pythagoras theorem (The theorem of hypotenuse). They gave importance to practical knowledge of making vedis

instead of the mathematical aspect. Sulbasutras give many sets of the measures of right angled triangles. We can find many sets in Apastambha Sulbasutras [6]. They are

1. If the sides of a rectangle are 3 and 4, then its diagonal is 5.

2. If the sides of a rectangle are 5 and 12, then its diagonal is 13.

3. If the sides of a rectangle are 8 and 15, then its diagonal is 17.

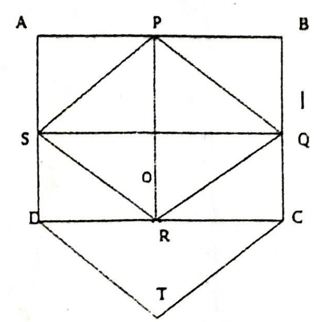
4. If the sides of a rectangle are 12 and 35, then its diagonal is 37.

These measures are used in the construction of vedis. All these can be used for the construction of Mahavedi. The following are some references of 'theorem of hypotenuse' in sulbasutras.

1. Construction of a square having an area of two times the area of a given square

In the case of Paitrki Vedi [7] (to be constructed in the Sakamedhaparvan of the caturmasya for the Mahapitryajna) one should construct a square having an area of two square purusas.

$CT = DT = 1$ purusa, where CT and DT are sides of a square, and CD its diagonal P, Q, R, S are the midpoints of AB, BC, CD, and DA. Square $PQRS$ is a square having an area one square purusa. The proof is as follows

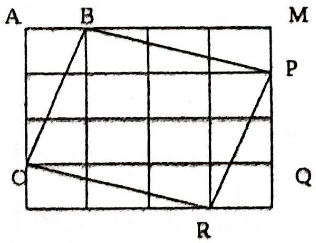


From figure

$$\text{Area of PBQO} = 2 \times \text{Area of triangle PBQ}$$
$$= 2 \times \text{Area of triangle POQ.}$$
$$= 2 \times \frac{1}{4} \text{ of the area of the square PQRS.}$$
$$= \frac{1}{2} \text{ of the area of square PQRS.}$$
$$\text{Area of square ABCD} = 4 \times \text{Area of the square PBQO.}$$
$$= 4 \times \frac{1}{2} \text{ Area of the square PQRS}$$
$$= 2 \times \text{Area of the square PQRS.}$$
$$= 2 \times \text{Area of the square produced by DT}$$

The figure is an indication that the square on the diagonal of a square is twice as large as that of the square.

2. Square produced on the diagonal is equal to the sum of the squares on the sides



Another reference to the theorem of hypotenuse is in Katyayana Sulbasutra. This says that in a right angled triangle [7] whose side is one pada and the other side is 3 padas, its hypotenuse will produce an area of ten square padas. In the Varunapraghas-aparvam of Caturmasya sacrifice, the northern altar is a square having an area of ten square padas.

Triangle ABC is a right angled triangle with one side one pada and the other side three padas. This is half of the rectangle with sides one pada and three padas respectively. The remaining three triangles are of the same size. Therefore the total area is $2 \times 3 = 6$ padas. So the rectangle BPCR is a square of area 10 padas.

From this it is clear that the square produced on this diagonal is equal to the sum of the squares produced on the sides. Some other indications of the same theorem are as follows,

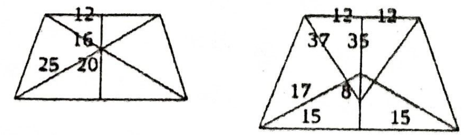
3. Some sets of the measures of the right angled triangles

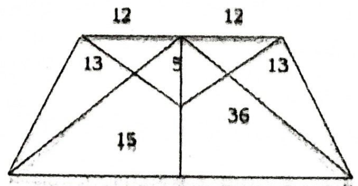
In a right angled triangle whose one side is two padas, and the other side six padas, the hypotenuse will produce an area of forty square padas [9].In a rectangle its diagonal produces by itself an area which its length and breadth produce separately [10].

In a square its diagonal produces an area which is double that of its side [11].

In Apastambha Sulbasutra, there is a reference of right angled triangles which are used in the construction of Mahavedi [12]. One of them says that the hypotenuse is 13 and the other sides are 5 and 12 units. The eastern corners are decided by making the sides three times the original length. Another one which is useful for fixing the western corners is such that the hypotenuse is 17,the length of whose sides is 8 and 15 units.

Another one for fixing the eastern corners is that the hypotenuse is 37, when the length of the sides is 12 and 35 units [13].In Apastambha Sulbasutra there is a reference to the right angled triangle whose sides are 3, 4 and 5 which is used for constructing Somavedi [14].There is a list of rightangled triangles in the Baudhayana Sulbasutra. The sides are listed as a) 3 and 4 b) 12 and 5 c) 15 and 8 d) 7 and 24 e) 12 and 35 f) 15 and 36. All these can be used for the construction of the Saumiki vedi or Mahavedi [15].





From these it is evident that ancient people in India were conscious about the construction of right angled triangles and the measurements of the hypotenuse and the other two sides.

4. Construction of a square having the area as the sum of areas of two squares.

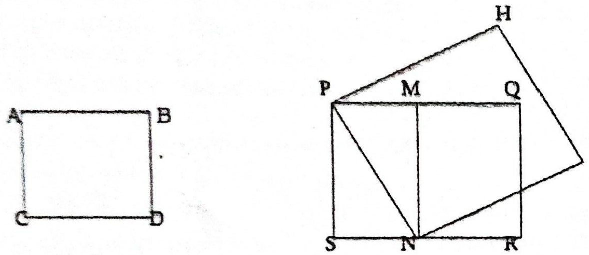
Nana pramanayo schaturasrayossamasa
Hrasiyasa karanya varsiyaso vridhra

Mullikheth vridhrasya ksnayarajjurubhe
Samasyati taduktam

(Apastambha Sulbasutram 2.4)

Make a rectangle on one side of the large square having length of one side equal to the smaller square. Then the area of the square made by a diagonal of this rectangle is equal to the sum of the areas of the above two squares.

Let ABCD be the small square and let PQRS be the larger square such that AB $= x$ and PQ $= y$



Let M and N are points on the sides PQ and RS such that PM $= x$ and SN $= x$. Consider the rectangle PMNS having sides x and y respectively.

Clearly PN be a diagonal of this rectangle. Construct a square having PN as a side, say PNGH.

$$PN^2 = PM^2 + MN^2 = x^2 + y^2$$

Area of the square PNGH

$= PN^2 = x^2 + Y^2$

$= AB^2 + PQ^2$

$=$ Area of the square ABCD $+$ Area of the square PQRS.

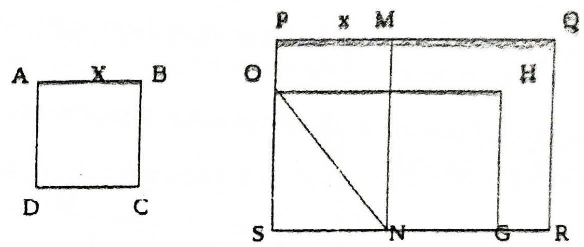
5. Construction of a square having area as the difference of areas of two squares

Caturasra caturasram
Nirjihirshan
Yavannirjihirsheth

Tasyakaranya
Varsiyaso vridhra
Mullikheth vridhrasya
Parsomanilmaksna
Yetarath parswa
Mupasamharet sa yatra
Nipatet tadapachindyat
Chinnaya nirustam

(Apastambha sullasutram 2–5)

Let ABCD be the small square and let PQRS be the larger square such that AB $= x$ and PQ $= y$



Mark two points M and N on the sides PQ and RS respectively such that PM $=$ SN $= x$. Let O be a point on the side PS such that NO $=$ NM. Construct a small square having OS as one side say OSGH. The area of the above two squares.

$$OS^2 = ON^2 - SN^2 = MN^2 - x^2 = y^2 - x^2$$

$\therefore$ Area of the square OSGH $= OS^2 = y^2 - x^2$.

Conclusion

It is clear that the constructions of all these vedis are not possible without the help of the knowledge of the proposition of the hypotenuse. The discovery of this proposition was before the period of Baudhayana Sulbasutra. The shape of the altars may differ but their areas will remain the same. The period of development of Geometry in India is before the time of Pythagoras.

Abbreviations

Br. Smt	-	Brhat Samhita
K.Sl	-	Katyayana Sulba sutra
B.Sl	:	Baudhayana Sulba sutra
Ap. Sl	-	Apastambha Slbhasutra