

Q.P Code D133862	Total Pages 2	Name 672123
		Register No.
THIRD SEMESTER UG DEGREE EXAMINATION, NOVEMBER 2025		
(CUFYUGP)		
MAT3CJ201 Multivariable Calculus		
2024 Admission Onwards		
Maximum Time :2 Hours		Maximum Marks :70

Section A

All Question can be answered. Each Question carries 3 marks (Ceiling: 24 Marks)

1	Let $x = t - 1, y = t + 1 : -\infty < t < \infty$, be the parametric equation and parameter interval for the motion of a particle in the xy-plane. Identify the particle's path by finding a Cartesian equation for it. Graph the Cartesian equation.
2	Replace the following polar equation by equivalent Cartesian equation, and identify their graphs. $r = \frac{4}{2 \cos \theta - \sin \theta}$
3	Find an equation for the level curve of the function $f(x, y, z) = \sqrt{x} - y - \ln z$ that passes through the point $(3, -1, 1)$.
4	Find $\lim_{(x,y) \rightarrow (1,1)} \frac{x^2 - 2xy + y^2}{x - y}; x \neq y$
5	Find $\frac{dw}{dt}$ at $t = 0$, if $w = x^2 + y^2, x = \cos t + \sin t, y = \cos t - \sin t$
6	Define local maxima and minima
7	Find the directional derivative of $f(x, y) = 4x^3y^2$ at $P = (2, 1)$ in the direction of $a = 4i - 3j$
8	Find $\int_0^{\pi/3} \int_0^{\cos y} x \sin y \, dx dy$
9	Find the average height of the paraboloid $z = x^2 + y^2$ over the square $0 \leq x \leq 2, 0 \leq y \leq 2$.
10	Change the Cartesian integral into an equivalent polar integral. Then evaluate the polar integral for the following. $\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2 + y^2) \, dx dy$

Section B

All Question can be answered. Each Question carries 6 marks (Ceiling: 36 Marks)

11	Find a plane through $P_0(2, 1, -1)$ and perpendicular to the line of intersection of the planes $2x + y - z = 3, x + 2y + z = 2$.
12	Sketch the surface $x = 4 - 4y^2 - z^2$
13	Show that the function $z = e^{-t} \sin(x/c)$ satisfies Heat Equation $\frac{\partial z}{\partial t} = c^2 \frac{\partial^2 z}{\partial x^2}$
14	Find the linearization of $f(x, y) = e^{2x-y}$ at the points $(0, 0)$. and $(0, 2)$
15	Find three positive numbers whose sum is 27 and such that the sum of their squares is as small as possible
16	Find the tangent plane and normal line of the surface $f(x, y, z) = x^2 + y^2 + z - 9 = 0$. at the point $P_0(1, 2, 4)$.
17	Find $\int_1^3 \int_x^{x^2} \int_0^{\ln z} x e^y dy dz dx$
18	Find the volume of the solid in the first octant bounded by the coordinate planes, the plane $x = 3$, and the parabolic cylinder $z = 4 - y^2$.

Section C

Answer any ONE. Each Question carries 10 marks (1x10=10 Marks)

19	(i) Find a spherical coordinate equation for the sphere $x^2 + y^2 + (z - 1)^2 = 1$. (ii) Find the equation of the plane through $(1, 1, -1)$, $(2, 0, 2)$, and $(0, -2, 1)$
20	Find the absolute maxima and minima of the function $f(x, y) = 2x^2 - 4x + yz - 4y + 1$ on the closed triangular plate bounded by the lines $x = 0, y = 2, y = 2x$ in the first quadrant.