D 112341 **(Pages : 2)** **Name.....**

Reg. No.....

FIRST SEMESTER (CUFYUGP) DEGREE EXAMINATION
NOVEMBER 2024

Mathematics

MAT 1CJ 101—DIFFERENTIAL CALCULUS

(2024 Admission onwards)

Time : Two Hours Maximum Marks : 70

Section A

All *questions can be answered.*
Each question carries 3 *marks.*
(Ceiling 24 *marks)*

1. Draw the graph of the equation $y^2 = 4x$.
2. Write any three properties of absolute value function $f(x) = |x|$.
3. Find $\lim_{x\to+\infty}\left(\frac{1-e^x}{1+e^x}\right)$.
4. Does $\lim_{x\to1}\frac{1}{1-x} = \lim_{x\to0^-}\frac{1}{x}$ exist ? Explain.
5. At what points are the function $\frac{x}{\sin x}$ continuous ?
6. Let $f(x) = x^2$ and $g(x) = x + 6$. Find $(g \circ f)(x)$ and $(g \circ f)'(x)$.
7. Find the absolute maximum and minimum values of $f(x) = 4x^2 - 12x + 10$ on the closed interval [1, 2].
8. Find the two x-intercepts of the function $f(x) = x^2 - 5x + 4$ and confirm that $f'(c) = 0$ at some point c between those intercepts.
9. Locate the critical points of the function $f(x) = x^3 - 3x^2$ and identify which critical points are stationary points.
10. Determine whether the statement "If the graph of f has a cusp at $x = 1$, then f cannot have an inflection point at $x = 1$" is true or false. Explain your answer.

Turn over

Section B

All *questions can be answered.*
Each question carries 6 *marks.*
(Ceiling 36 *marks)*

11. Find the center and radius of the circle $x^2 + y^2 + 4x - 6y - 3 = 0$.

12. Determine whether the statement "If $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist, then so does $\lim_{x \to a} [f(x) + g(x)]$" is true or false. Explain your answer.

13. Define $h(2)$ in a way that extends $h(t) = \dfrac{t^2 + 3t - 10}{(t - 2)}$ to be continuous at $t = 2$.

14. Find $\dfrac{dy}{dx}$ if $2y = x^2 + \sin y$.

15. Find the absolute extrema of $f(x) = \sqrt{x} + \cos x$ on $\{0, 2\pi]$.

16. Find the interval on which the function $f(x) = ax^2 + bx + c, a \neq 0$, is increasing and decreasing. Describe the reasoning bahind your answer.

17. Find the asymptotes of the graph of $f(x) = -\dfrac{8}{x^2 - 4}$.

18. Solve $\lim_{x \to \infty} \dfrac{2\sqrt{x} + x^{-1}}{3x - 4}$.

Section C

Answer any **one** *question.*
The question carries 10 *marks.*

19. (a) Suppose $\lim_{x \to 1} f(x) = 2$ and $\lim_{x \to 1} g(x) = 9$. Find $\lim_{x \to 1} \dfrac{f(x) - x^2 g(x)}{x - f(x)}$.

    (b) Explain the continuity of the function $f(x) = \dfrac{3x + 4}{x^2 - 4}$.

20. (a) Suppose that $f(-1) = 3$ and $f'(x) = 0$ for all x. Must $f(x) = 3$ for all x ? Give reasons for your answer.

    (b) Discuss the concavity and convexity of the curve $y = x^2$.

(1 × 10 = 10 marks)